COMP2804 – Winter 2020 – Assignment 2 – Andy Chia – 101111058

**Arrangements of PUNKEYDOODLES**

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| 2. | How may distinct ways are there to rearrange the letters in PUNKEYDOODLES (the name of a town in Ontario)?  P – 1  U – 1  N – 1  K – 1  E – 2  Y – 1  D – 2  O – 2  L – 1  S – 1   1. Chose 1 position for P from 13 positions 2. Chose 1 position for U from 12 positions 3. Chose 1 position for N from 11 positions 4. Chose 1 position for K from 10 positions 5. Chose 2 positions for E from 9 positions 6. Chose 1 position for Y from 7 positions 7. Chose 2 positions for D from 6 positions 8. Chose 2 positions for O from 4 positions 9. Chose 1 position for L from 2 positions 10. Make the last position S |

**Modern Coupling**

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| 3. | 1. | Suppose we have a group of n women and n men, all heterosexual and all strictly monogamous. How many ways are there to make n couples out of these 2n people?   * n men * n women * Heterosexual And all monogamous * # men = # women  1. first man chooses out of n women 2. second man chooses out of n - 1 women 3. repeat for each man |
|  | 2. | Suppose we have 2n people, each of whom is bisexual but strictly monogamous. How many ways are there to make n couples out of these 2n people?   * 2n people * bisexual  1. First person chooses out of n – 1 people (the one missing being the picker) 2. Third person chooses out of n – 3 people 3. Fifth person chooses out of n – 5 people 4. repeat for every next person 5. Second last person chooses last person (sorry) |

**Pigeonholing**

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| 4. | 1. | In an anonymous survey of a group of 20 men and 20 women, the men reported a total of 81 sexual encounters with women in the group, and all women reported having at most 4 sexual encounters with men in the group. Is everyone telling the truth?   * 20 men and 20 women * 81 total sexual encounters from men with the women * women having at most 4 sexual encounters with men   Suppose every woman had sexual encounters 4 times being the most amount  Since there are more reported sexual encounters from men than the maximum amount of sexual encounter that women can have added together, this is false.  By PHP there must have been one woman that has had a sexual intercourse times (5) to match the number of reported sexual encounters by the men |
|  | 2. | A group of n agents all start at the same location and each one takes a ≤m-walk on the line, where a ≤m-walk is a sequence of at most m steps and each step moves the agent one unit to the left or one unit to the right. (Different agents might take different walks.) Prove that, if n>2m+1, then some pair of agents finishes their walk at the same location.   * n = # of agents * m = # of steps * Each agent travels at most m steps * Agents either move to the left or the right * if then some pair of agents finish at the same location   n = agents -> pigeons  l = locations -> holes  if n > l then PHP  show  since you can only take left and right steps [-m, m]  at most means that the agents can stop an infinite distance  |l| = 2m + 1 because m to the left, m to the right and its initial position |
|  | 3. | A group of n agents all start at the same location and each one takes an m-walk on the line, where a m-walk is a sequence of exactly m steps and each step moves the agent one unit to the left or one unit to the right. (Different agents might take different walks.) Prove that, if n>m+1, then some pair of agents finishes their walk at the same location.   * n = # of agents * m = # of steps * All agents travel m steps exactly * Agents either move to the left or the right * Prove that, if n>m+1, then some pair of agents finishes their walk at the same location.   n = agents -> pigeons  l = locations -> holes  if n > l then PHP  show  Since you can only take left and right steps [-m, m]  |l| = m + 1 because all agents walk the same distance and initial starting position |
|  | 4. | Let V = {v1, …, vk} be any set of vectors in R2. Suppose n agents each start at (0,0) and each takes a mV-walk where a mV-walk consists of a sequence of exactly m steps and each step moves the agent along a vector in V. Prove that, if n>(m+k−1 k−1), then some pair of agents finishes their walk at the same location.   * n = agents -> pigeons * p = position -> holes * m = steps * Prove that, if , then some pair of agents finishes their walk at the same location.   if n > p then PHP  show  M being a step in a direction and k being the possible vectors in the direction that the agent walks  because all agents walk the same distance and they go all in the possible k directions |
|  | 5. | Let S be a k-element subset of {1,…,n}. Prove that, if k>⌈n/2⌉, then there exists x,y∈S such that x−y=1   * S = {1, …, n} * Prove that, if , then there exist such that   x and y are in s  x > y, x = y + 1  If more than half there exist two numbers following each other  Since there are more than half the amount of numbers, there must be a number between two numbers that will follow each other to have all the numbers  If PHP |
|  | 6. | Let S be a k-element subset of {1,…,n}. Prove that, if (k 2)>n−1, then there exists a,b,x,y∈S such that a≠b, {a,b}≠{x,y} and b−a=y−x. (Note that it may be the case that b=x or a=y.)   * a,b,x,y are in S * a is not equal to b * {a,b} cant have the same numbers as {x,y} * b and a cant have the same gap as x and y * b and be equal to x or a can be equal to y * Prove k choses 2 > n – 1 |
|  | 7. |  |

**Recurrences**

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| 5. | 1. | N = occurrences  a = end (0)  b = number to f(n)  Proof:  A = 5  B = 8  N = 3  Base case:  N=0  Induction: |
|  | 2. | ½ = Ending (0)  n = occurrences & to be multiplied  ½ = to be multiplied every time  Proof  N = 3  Base case:  N=0  Induction: |
|  | 3. | 4 times the occurrences of 3 in n  Proof: n = 10  Base case:  n = 1  Induction: |
|  | 4. | If n = 0 or n = 1 then cc appears 0 times  If n = 2 then cc appears 1 time  If n = 3 then cc appears 7 times (1 \* 3) + 4   * 1 time from ac (A2) * 1 time from bc (A2) * 4 time from cc (A2) * 1 time from dc(A2)   If n = 4 then cc appears 40 times (7 \* 3) + 16 + (1 \* 3)  If n = 5 then cc appears 205 times (40 \* 3) + 64 + (7\*3)  b(n) = {0 if n = 0  0 if n = 1  if n >= 2 |
|  | 5. | a.  Base case:  n = 3  b. |